

Question 6

Find the area of the portion of

$$z = 4 - x - y$$

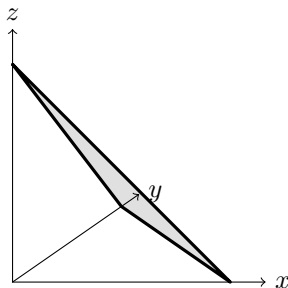
in the first octant.

Tasks

1. Determine the projection in the xy -plane.
2. Evaluate the graph surface-area integral.
3. Verify using the three intercept vertices.

Question 6 – Solution

Strategy. The coordinate-plane restrictions cut a triangular patch from a plane with constant slope.

Step 1: Region

In the first octant, $x \geq 0$, $y \geq 0$, and $z = 4 - x - y \geq 0$. Thus the projection is

$$D = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq 4 - x\},$$

a right triangle of area 8.

Step 2: Surface integral Since $f_x = f_y = -1$,

$$S = \iint_D \sqrt{1 + (-1)^2 + (-1)^2} dA = \sqrt{3}(8) = \boxed{8\sqrt{3}}.$$

Verification From $(4, 0, 0)$, use edge vectors $\mathbf{a} = \langle -4, 4, 0 \rangle$ and $\mathbf{b} = \langle -4, 0, 4 \rangle$. Then $|\mathbf{a} \times \mathbf{b}| = 16\sqrt{3}$, so the triangle area is $\frac{1}{2}(16\sqrt{3}) = 8\sqrt{3}$.