

Question 5

Find the area of the sideways paraboloid

$$x = y^2 + z^2$$

over the disk $y^2 + z^2 \leq 1$ in the yz -plane.

Tasks

1. State the surface-area formula for a graph $x = g(y, z)$.
2. Evaluate using polar coordinates in the yz -plane.
3. Check the result against the projected disk area.

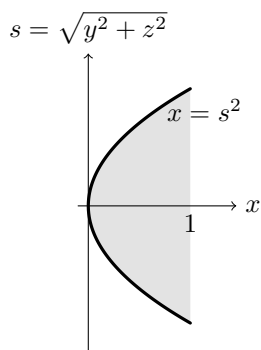
Question 5 – Solution

Strategy. Project onto the yz -plane, where both the region and squared gradient are radial.

Step 1: Projection formula For $x = g(y, z)$,

$$dS = \sqrt{1 + g_y^2 + g_z^2} dy dz.$$

Here $g = y^2 + z^2$, so with $s^2 = y^2 + z^2$ the factor is $\sqrt{1 + 4s^2}$.



Step 2: Evaluate

$$\begin{aligned} S &= \int_0^{2\pi} \int_0^1 \sqrt{1 + 4s^2} s ds d\theta \\ &= 2\pi \left[\frac{(1 + 4s^2)^{3/2}}{12} \right]_0^1 = \boxed{\frac{\pi}{6} (5\sqrt{5} - 1)}. \end{aligned}$$

Verification The graph projects one-to-one onto a disk of area π , and the area factor exceeds 1 except at its center. Accordingly, the computed surface area is strictly larger than π .