

Question 3

Find the area of the conical surface

$$z = \sqrt{x^2 + y^2}$$

lying above the annulus $1 \leq x^2 + y^2 \leq 9$.

Tasks

1. Explain why the graph formula is valid on the annulus.
2. Evaluate the area.
3. Verify with the lateral-area formula for a conical frustum.

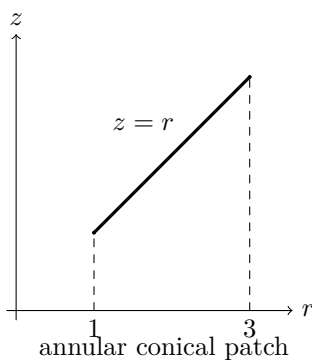
Question 3 – Solution

Strategy. The annulus avoids the cone's nondifferentiable vertex, and the cone has constant slope there.

Step 1: Gradient Write $r = \sqrt{x^2 + y^2}$. For $r > 0$,

$$f_x = \frac{x}{r}, \quad f_y = \frac{y}{r}, \quad f_x^2 + f_y^2 = 1.$$

The entire projection has $r \geq 1$, so no derivative singularity occurs.



Step 2: Area

$$S = \iint_D \sqrt{1 + 1} \, dA = \sqrt{2} \pi (3^2 - 1^2) = \boxed{8\sqrt{2} \pi}.$$

Verification The frustum has radii 1 and 3. Its slant height is $\sqrt{(3-1)^2 + (3-1)^2} = 2\sqrt{2}$, so the classical lateral area $\pi(R+r)\ell = \pi(3+1)(2\sqrt{2}) = 8\sqrt{2}\pi$ agrees.