

Question 2

Find the area of the paraboloid

$$z = x^2 + y^2$$

above the disk $x^2 + y^2 \leq 4$.

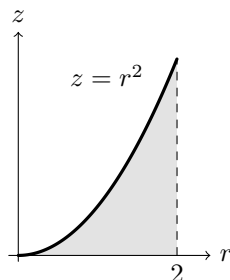
Tasks

1. Form the graph surface-area integrand.
2. Use polar coordinates to evaluate the integral.
3. Check the answer against the projected area.

Question 2 – Solution

Strategy. The squared gradient depends only on r , so polar coordinates reduce the area integral to one variable.

Step 1: Geometry and gradient



For $f = x^2 + y^2$,

$$f_x = 2x, \quad f_y = 2y, \quad \sqrt{1 + f_x^2 + f_y^2} = \sqrt{1 + 4r^2}.$$

Step 2: Evaluate

$$\begin{aligned} S &= \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2} \, r \, dr \, d\theta \\ &= 2\pi \left[\frac{(1 + 4r^2)^{3/2}}{12} \right]_0^2 = \boxed{\frac{\pi}{6} (17\sqrt{17} - 1)}. \end{aligned}$$

Verification The area factor is at least 1, so S must exceed the projected disk area 4π . The exact result does. Its integrand also increases with r , matching the increasing steepness of the paraboloid.