

Question 1

Find the area of the part of the plane

$$z = 2x - y$$

lying above the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$.

Tasks

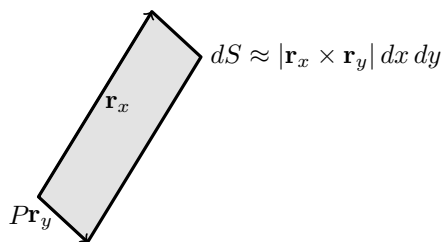
1. Derive the graph surface-area factor from tangent vectors.
2. Evaluate the surface area.
3. Verify by interpreting the constant area scale.

Question 1 – Solution

Strategy. Parametrize the graph by its projection and use the tangent-parallelogram area scale.

Step 1: Derive the element For a graph $z = f(x, y)$,

$$\mathbf{r}(x, y) = \langle x, y, f(x, y) \rangle, \quad \mathbf{r}_x = \langle 1, 0, f_x \rangle, \quad \mathbf{r}_y = \langle 0, 1, f_y \rangle.$$



Thus

$$|\mathbf{r}_x \times \mathbf{r}_y| = |\langle -f_x, -f_y, 1 \rangle| = \sqrt{1 + f_x^2 + f_y^2}.$$

Step 2: Apply to the plane Here $f_x = 2$ and $f_y = -1$, so

$$S = \int_0^3 \int_0^2 \sqrt{1 + 2^2 + (-1)^2} dy dx = \sqrt{6}(3)(2) = \boxed{6\sqrt{6}}.$$

Verification Every small projected area is enlarged by the same factor $\sqrt{6}$. Since the rectangle has area 6, the surface area must be $6\sqrt{6}$.