

Question 9

For $a, b > 0$, let

$$E_{a,b} = \left\{ (x, y) : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1 \right\}.$$

Suppose

$$\text{area}(E_{a,b}) = 6\pi, \quad \iint_{E_{a,b}} x^2 \, dA = \frac{27\pi}{2}.$$

Tasks

1. Use $x = au$, $y = bv$ to translate both measurements.
2. Recover a and b .
3. Verify both original conditions.

Question 9 – Solution

Strategy. Scale the ellipse to the unit disk; area determines ab , while the x^2 moment determines one additional power of a^2 .

Step 1: Area condition The map $x = au$, $y = bv$ has $dA = ab \, du \, dv$. Since the unit disk has area π ,

$$\pi ab = 6\pi, \quad \text{so} \quad ab = 6.$$

Step 2: Moment condition On the unit disk,

$$\iint_D u^2 \, du \, dv = \frac{\pi}{4}$$

by symmetry or a polar calculation. Therefore

$$\iint_{E_{a,b}} x^2 \, dA = a^2(ab) \frac{\pi}{4} = \frac{3\pi a^2}{2}.$$

Equating this with $27\pi/2$ gives $a^2 = 9$. Positivity then yields

$$\boxed{a = 3}, \quad \boxed{b = 2}.$$

Verification The recovered ellipse has area $\pi(3)(2) = 6\pi$. Its moment is $a^2(ab)\pi/4 = (9)(6)\pi/4 = 27\pi/2$, so both measurements are satisfied without sign ambiguity.