

Question 7

Let

$$R = \{(x, y) : 0 \leq x \leq 2, x^2 \leq y \leq x^2 + 3\}.$$

Use $u = x$ and $v = y - x^2$ to evaluate

$$\iint_R (y - x^2) dA.$$

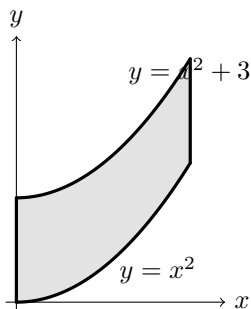
Tasks

1. Show how the change of variables straightens the parabolic strip.
2. Compute the Jacobian and integral.
3. Verify directly from the vertical thickness.

Question 7 – Solution

Strategy. Measure vertical displacement from the lower parabola; the curved strip then becomes a rectangle.

Step 1: Straighten the region



The inverse map is

$$x = u, \quad y = u^2 + v,$$

and the bounds become $0 \leq u \leq 2$, $0 \leq v \leq 3$.

Step 2: Jacobian and integral

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 0 \\ 2u & 1 \end{vmatrix} = 1.$$

Because $y - x^2 = v$,

$$I = \int_0^2 \int_0^3 v \, dv \, du = (2) \left[\frac{v^2}{2} \right]_0^3 = \boxed{9}.$$

Verification For each fixed x , the displacement $t = y - x^2$ ranges from 0 to 3. Thus direct integration gives $\int_0^2 \int_0^3 t \, dt \, dx = 9$. The unit Jacobian reflects a vertical shear with no area scaling.