

Question 5

Let

$$R = \{(x, y) : 1 \leq x \leq 3, x \leq y \leq 2x\}.$$

Use $u = x$ and $v = y/x$ to evaluate

$$\iint_R \frac{1}{x} dA.$$

Tasks

1. Find the inverse map and transformed bounds.
2. Compute the Jacobian and evaluate.
3. Verify directly in Cartesian coordinates.

Question 5 – Solution

Strategy. The ratio $v = y/x$ turns the two sloping sides into constant-coordinate boundaries.

Step 1: Inverse and bounds Since $x > 0$ on R ,

$$x = u, \quad y = uv.$$

The region becomes the rectangle

$$1 \leq u \leq 3, \quad 1 \leq v \leq 2.$$

The inverse map is one-to-one because $u = x$ and $v = y/x$ recover each point uniquely.

Step 2: Jacobian

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} 1 & 0 \\ v & u \end{vmatrix} = u.$$

Thus

$$I = \int_1^3 \int_1^2 \frac{1}{u}(u) \, dv \, du = \int_1^3 \int_1^2 1 \, dv \, du = \boxed{2}.$$

Verification Direct Cartesian integration gives

$$\int_1^3 \int_x^{2x} \frac{1}{x} \, dy \, dx = \int_1^3 \frac{2x - x}{x} \, dx = 2.$$

The two methods agree exactly, including the variable area scale u .