

Question 4

Let R be bounded by the four lines

$$x + 2y = 0, \quad x + 2y = 2, \quad 3x - y = 1, \quad 3x - y = 4.$$

Use $u = x + 2y$, $v = 3x - y$ to evaluate

$$\iint_R (x + 2y)(3x - y) \, dA.$$

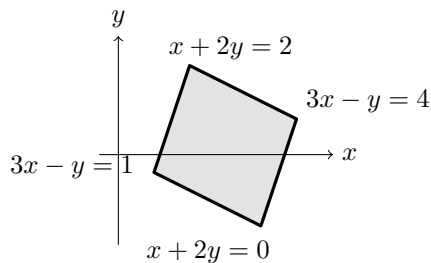
Tasks

1. Identify the transformed rectangle.
2. Compute the Jacobian and integral.
3. Verify that the transformation preserves one-to-one coverage.

Question 4 – Solution

Strategy. Each pair of parallel boundary lines becomes a pair of constant-coordinate sides.

Step 1: Geometry



The image is the rectangle $0 \leq u \leq 2$, $1 \leq v \leq 4$. The coefficient matrix has determinant

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix} = -7.$$

Since this determinant is nonzero, the linear map is globally one-to-one.

Step 2: Change variables We have $dA = \frac{1}{7} du dv$ and the integrand is uv . Hence

$$\begin{aligned} I &= \frac{1}{7} \int_0^2 \int_1^4 uv \, dv \, du \\ &= \frac{1}{7} \left[\frac{u^2}{2} \right]_0^2 \left[\frac{v^2}{2} \right]_1^4 = \boxed{\frac{15}{7}}. \end{aligned}$$

Verification On the transformed rectangle, $u \geq 0$ and $v > 0$, so the positive answer has the required sign. The constant area scale $1/7$ also matches the inverse determinant.