

Question 3

Let

$$E = \left\{ (x, y) : \frac{x^2}{9} + \frac{y^2}{4} \leq 1 \right\}.$$

Use $x = 3u$, $y = 2v$ and then polar coordinates in the uv -plane to evaluate

$$\iint_E \left(\frac{x^2}{9} + \frac{y^2}{4} \right) dA.$$

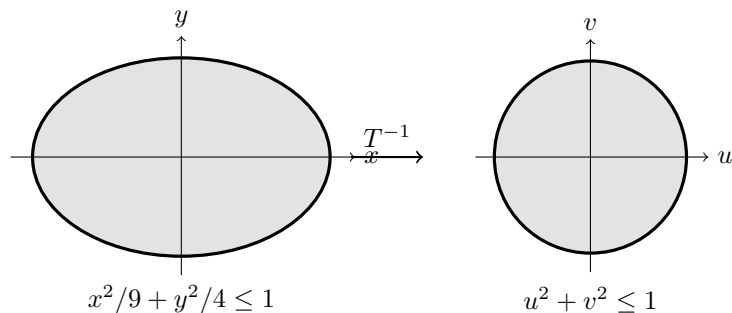
Tasks

1. Map the ellipse to a unit disk.
2. Track both Jacobian factors and evaluate.
3. Verify by comparing the integral with the ellipse area.

Question 3 – Solution

Strategy. First remove the unequal axis scales, then exploit radial symmetry on the resulting unit disk.

Step 1: Linear scaling



The map $x = 3u$, $y = 2v$ sends the unit disk D to E , with

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = 6, \quad \frac{x^2}{9} + \frac{y^2}{4} = u^2 + v^2.$$

Step 2: Polar coordinates in D Write $u = r \cos \theta$, $v = r \sin \theta$ and $du dv = r dr d\theta$. Then

$$\begin{aligned} I &= 6 \int_0^{2\pi} \int_0^1 r^2 (r dr d\theta) \\ &= 6(2\pi) \left[\frac{r^4}{4} \right]_0^1 = \boxed{3\pi}. \end{aligned}$$

Verification The ellipse area is 6π , and the transformed radial-square function has average $1/2$ on the unit disk. Thus the integral must be $(6\pi)(1/2) = 3\pi$.