

Question 2

In the first quadrant, let R be bounded by

$$xy = 1, \quad xy = 4, \quad \frac{x}{y} = 1, \quad \frac{x}{y} = 9.$$

Use $u = xy$ and $v = x/y$ to evaluate $\iint_R (x/y) \, dA$.

Tasks

1. Show that the transformation is one-to-one on R .
2. Find the inverse Jacobian.
3. Evaluate and check the result against the integrand range.

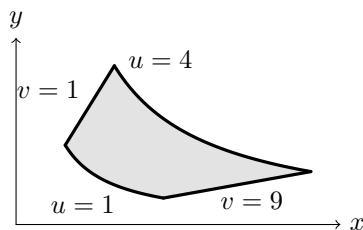
Question 2 – Solution

Strategy. Product and ratio coordinates straighten the hyperbolas and rays into a rectangle.

Step 1: Region and inverse In the first quadrant,

$$x = \sqrt{uv}, \quad y = \sqrt{\frac{u}{v}},$$

so positive (u, v) determine a unique (x, y) . The image is $1 \leq u \leq 4$, $1 \leq v \leq 9$.



Step 2: Jacobian Differentiating the inverse formulas gives

$$\left| \frac{\partial(x, y)}{\partial(u, v)} \right| = \frac{1}{2v}.$$

Therefore the factor $x/y = v$ cancels the v in the denominator.

Step 3: Evaluate

$$I = \int_1^4 \int_1^9 v \left(\frac{1}{2v} \right) dv du = \frac{1}{2}(3)(8) = \boxed{12}.$$

Verification The region area is $\int_1^4 \int_1^9 (2v)^{-1} dv du = 3 \ln 3$. Since $1 \leq x/y \leq 9$, the bounds $3 \ln 3 \leq I \leq 27 \ln 3$ hold; $I = 12$ lies between them.