

Question 1

Let

$$R = \{(x, y) : |x + y| \leq 2, |x - y| \leq 1\}.$$

Use $u = x + y$ and $v = x - y$ to evaluate

$$\iint_R (x + y)^2 dA.$$

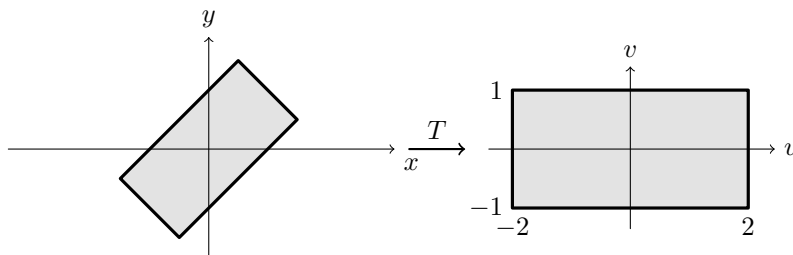
Tasks

1. Describe the image of R in the uv -plane.
2. Compute the inverse Jacobian and evaluate the integral.
3. Verify the sign and scale of the answer.

Question 1 – Solution

Strategy. The boundary expressions themselves become coordinates, turning the tilted rectangle into an axis-aligned rectangle.

Step 1: Transform the region



The inequalities become $-2 \leq u \leq 2$ and $-1 \leq v \leq 1$. Solving $u = x + y$, $v = x - y$ gives

$$x = \frac{u+v}{2}, \quad y = \frac{u-v}{2}.$$

Step 2: Jacobian

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}, \quad dA = \frac{1}{2} du dv.$$

Step 3: Evaluate

$$\begin{aligned} I &= \int_{-2}^2 \int_{-1}^1 u^2 \left(\frac{1}{2} \right) dv du \\ &= \frac{1}{2}(2) \left[\frac{u^3}{3} \right]_{-2}^2 = \boxed{\frac{16}{3}}. \end{aligned}$$

Verification The integrand is nonnegative. The transformed rectangle has area 8, scaled by $1/2$, so $\text{area}(R) = 4$; the result corresponds to average value $4/3$, which is below the maximum $u^2 = 4$.