

Question 4

Let E be the upper half of the ball $x^2 + y^2 + z^2 \leq 9$.

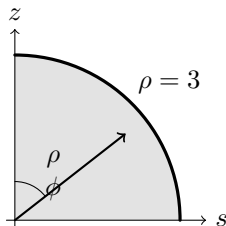
Tasks

1. Use spherical coordinates to compute $\iiint_E z \, dV$.
2. Find the average height z_{avg} .
3. Check its geometric range.

Question 4 – Solution

Strategy. Write $z = \rho \cos \phi$ and use the upper-hemisphere bounds $0 \leq \phi \leq \pi/2$.

Step 1: Geometry and integral



$$\begin{aligned} I &= \int_0^{2\pi} \int_0^{\pi/2} \int_0^3 (\rho \cos \phi) \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \\ &= (2\pi) \left[\frac{\sin^2 \phi}{2} \right]_0^{\pi/2} \left[\frac{\rho^4}{4} \right]_0^3 = \boxed{\frac{81\pi}{4}}. \end{aligned}$$

Step 2: Average height The hemisphere volume is $V = \frac{1}{2} \frac{4\pi}{3} (3^3) = 18\pi$. Thus

$$\boxed{z_{\text{avg}} = \frac{I}{V} = \frac{9}{8}}.$$

Verification Every point has $0 \leq z \leq 3$, and $9/8$ lies in that interval. The value is below the mid-height $3/2$ because cross-sectional area is greatest near $z = 0$.