

Question 8

Let E be the solid between the paraboloid $z = x^2 + y^2$ and the cone $z = 2\sqrt{x^2 + y^2}$.

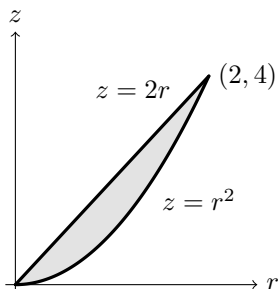
Tasks

1. Determine the cylindrical bounds.
2. Compute $\iiint_E z \, dV$.
3. Find the average value of z on E .

Question 8 – Solution

Strategy. Find the nonzero intersection radius, then integrate z between the radial graphs.

Step 1: Geometry The surfaces meet where $r^2 = 2r$, so $r = 0$ or $r = 2$. On $0 \leq r \leq 2$, the cone lies above the paraboloid.



Thus $0 \leq \theta \leq 2\pi$, $0 \leq r \leq 2$, and $r^2 \leq z \leq 2r$.

Step 2: Integral of z

$$\begin{aligned} I &= \int_0^{2\pi} \int_0^2 \int_{r^2}^{2r} zr \, dz \, dr \, d\theta \\ &= 2\pi \int_0^2 \left(2r^3 - \frac{1}{2}r^5 \right) dr = \boxed{\frac{16\pi}{3}}. \end{aligned}$$

Step 3: Average The volume is

$$V = 2\pi \int_0^2 (2r - r^2)r \, dr = \frac{8\pi}{3}.$$

Therefore $\boxed{z_{\text{avg}} = I/V = 2}$. This lies between the solid's minimum and maximum heights, 0 and 4.