

Question 6

The cone

$$0 \leq z \leq 3 - \sqrt{x^2 + y^2}$$

has constant density $\rho = 2$.

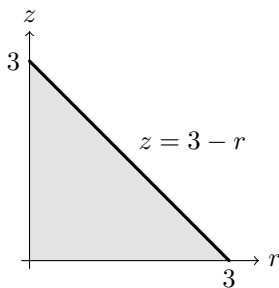
Tasks

1. Find its mass.
2. Find its moment of inertia I_z about the z -axis.
3. Find its radius of gyration $k_z = \sqrt{I_z/M}$.

Question 6 – Solution

Strategy. Use $x^2 + y^2 = r^2$ for the squared distance to the axis and $dV = r \, dz \, dr \, d\theta$.

Step 1: Region The meridian triangle gives $0 \leq r \leq 3$ and $0 \leq z \leq 3 - r$.



Step 2: Mass

$$M = \int_0^{2\pi} \int_0^3 \int_0^{3-r} 2r \, dz \, dr \, d\theta = 4\pi \int_0^3 r(3-r) \, dr = \boxed{18\pi}.$$

Step 3: Moment and radius of gyration

$$I_z = \iiint_E (x^2 + y^2) \rho \, dV = 4\pi \int_0^3 r^3(3-r) \, dr = \boxed{\frac{243\pi}{5}},$$
$$k_z = \sqrt{\frac{I_z}{M}} = \boxed{\sqrt{\frac{27}{10}}}.$$

Verification $k_z \approx 1.64$ lies between the minimum and maximum distances 0 and 3 from the axis. Also $M = 2 \cdot \frac{1}{3}\pi(3^2)(3) = 18\pi$ agrees with the cone-volume formula.