

Question 5

A solid annular cylinder is described by

$$1 \leq x^2 + y^2 \leq 4, \quad 0 \leq z \leq 3,$$

and has density $\rho(x, y, z) = \sqrt{x^2 + y^2} + z$.

Tasks

1. Find its mass using cylindrical coordinates.
2. Find its average density.
3. Compare the average with the density range.

Question 5 – Solution

Strategy. The annular cylinder becomes a rectangular box in (r, θ, z) , while the density becomes $r + z$.

Step 1: Mass The bounds are $0 \leq \theta \leq 2\pi$, $1 \leq r \leq 2$, and $0 \leq z \leq 3$. Therefore

$$\begin{aligned} M &= \int_0^{2\pi} \int_1^2 \int_0^3 (r + z)r \, dz \, dr \, d\theta \\ &= 2\pi \int_1^2 \left(3r^2 + \frac{9}{2}r \right) dr = 2\pi \left[r^3 + \frac{9}{4}r^2 \right]_1^2 = \boxed{\frac{55\pi}{2}}. \end{aligned}$$

Step 2: Average density The volume is $\pi(2^2 - 1^2)(3) = 9\pi$. Hence

$$\boxed{\rho_{\text{avg}} = \frac{M}{V} = \frac{55}{18}}.$$

Verification On the solid, $1 \leq r + z \leq 5$, and $55/18 \approx 3.06$ lies strictly in that range. The units of M/V are density units, as required.