

Question 4

Let

$$E = \{(x, y, z) : x^2 + y^2 \leq 4x, 0 \leq z \leq \sqrt{x^2 + y^2}\}.$$

Find the volume of E using cylindrical coordinates.

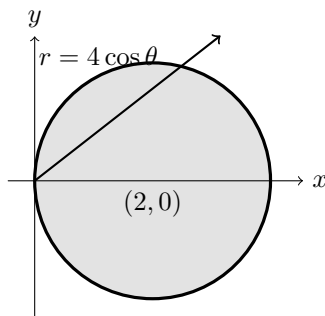
Tasks

1. Convert the offset cylindrical boundary and determine a nonduplicating angular interval.
2. Evaluate the volume.
3. Check the radial bounds at the endpoint and central rays.

Question 4 – Solution

Strategy. Convert the footprint to $r \leq 4 \cos \theta$ and use the conical height $z = r$.

Step 1: Footprint From $r^2 \leq 4r \cos \theta$, nonnegative radii satisfy $0 \leq r \leq 4 \cos \theta$. This requires $-\pi/2 \leq \theta \leq \pi/2$ and describes the disk centered at $(2, 0)$ with radius 2.



Step 2: Evaluate

$$\begin{aligned} V &= \int_{-\pi/2}^{\pi/2} \int_0^{4 \cos \theta} \int_0^r r \, dz \, dr \, d\theta \\ &= \frac{64}{3} \int_{-\pi/2}^{\pi/2} \cos^3 \theta \, d\theta = \frac{64}{3} \left(\frac{4}{3} \right) = \boxed{\frac{256}{9}}. \end{aligned}$$

Verification At $\theta = \pm\pi/2$ the radial interval collapses to $r = 0$; at $\theta = 0$ it reaches $r = 4$. These are exactly the leftmost and rightmost points of the footprint, so no rays are omitted or duplicated.