

Question 3

Let E lie above the cone $z = \sqrt{x^2 + y^2}$ and below the paraboloid $z = 2 - x^2 - y^2$.

Tasks

1. Find the curve where the two surfaces meet.
2. Set up and evaluate $\iiint_E 1 \, dV$ in cylindrical coordinates.
3. Verify that the vertical bounds are correctly ordered.

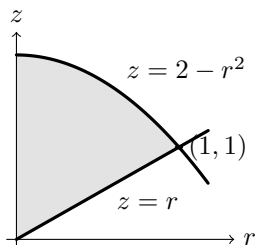
Question 3 – Solution

Strategy. Convert both surfaces to functions of r , solve for their positive intersection radius, and integrate top minus bottom.

Step 1: Intersection The surfaces are $z = r$ and $z = 2 - r^2$. Their intersection satisfies

$$r = 2 - r^2 \iff (r - 1)(r + 2) = 0.$$

Since $r \geq 0$, the boundary circle is $r = 1, z = 1$.



Step 2: Integral

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^1 \int_r^{2-r^2} r \, dz \, dr \, d\theta \\ &= 2\pi \int_0^1 (2 - r^2 - r)r \, dr = 2\pi \left(1 - \frac{1}{4} - \frac{1}{3} \right) = \boxed{\frac{5\pi}{6}}. \end{aligned}$$

Verification For $0 \leq r \leq 1$, $2 - r^2 - r = (1 - r)(r + 2) \geq 0$, so the upper and lower z -bounds never reverse.