

Question 2

Find the volume inside the paraboloid

$$z = 9 - x^2 - y^2$$

and above the xy -plane.

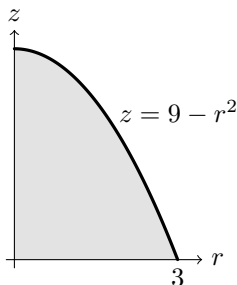
Tasks

1. Determine the cylindrical bounds from the surface intersection.
2. Evaluate the volume.
3. Check the answer against a containing cylinder.

Question 2 – Solution

Strategy. In cylindrical coordinates the paraboloid becomes $z = 9 - r^2$; its intersection with $z = 0$ determines the radial endpoint.

Step 1: Geometry Setting $z = 0$ gives $r = 3$. A meridian half-plane shows the height above each radius.



Thus $0 \leq \theta \leq 2\pi$, $0 \leq r \leq 3$, and $0 \leq z \leq 9 - r^2$.

Step 2: Evaluate

$$\begin{aligned} V &= \int_0^{2\pi} \int_0^3 \int_0^{9-r^2} r \, dz \, dr \, d\theta \\ &= 2\pi \int_0^3 (9r - r^3) \, dr = 2\pi \left[\frac{9}{2}r^2 - \frac{1}{4}r^4 \right]_0^3 = \boxed{\frac{81\pi}{2}}. \end{aligned}$$

Verification The solid lies inside the cylinder of radius 3 and height 9, whose volume is 81π . The computed volume is positive and exactly half that containing-cylinder volume.