

Question 1

Let

$$E = \{(x, y, z) : x^2 + y^2 \leq 4, x \geq 0, y \geq 0, -1 \leq z \leq 2\}.$$

Evaluate $\iiint_E (x^2 + y^2) dV$ using cylindrical coordinates.

Tasks

1. Convert the region, integrand, and volume element.
2. Evaluate the cylindrical integral.
3. Verify the result using an average value.

Question 1 – Solution

Strategy. Treat the solid as a quarter-cylinder; radial symmetry makes the integrand r^2 .

Step 1: Convert Cylindrical coordinates give

$$x = r \cos \theta, \quad y = r \sin \theta, \quad x^2 + y^2 = r^2, \quad dV = r \, dz \, dr \, d\theta.$$

The first-quadrant quarter-cylinder has bounds

$$0 \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq r \leq 2, \quad -1 \leq z \leq 2.$$

Step 2: Evaluate

$$\begin{aligned} I &= \int_0^{\pi/2} \int_0^2 \int_{-1}^2 r^2 (r \, dz \, dr \, d\theta) \\ &= \left(\frac{\pi}{2}\right) (3) \left[\frac{r^4}{4}\right]_0^2 = \boxed{6\pi}. \end{aligned}$$

Verification The volume is $(\pi 2^2/4)(3) = 3\pi$. On a radius-2 disk, the average of r^2 is $2^2/2 = 2$; restricting to one quadrant does not change that radial average. Thus $I = (3\pi)(2) = 6\pi$.