

Question 10

A solid E has volume 12, and $2 \leq f \leq 7$ throughout E . Another integrable function g satisfies $|g - f| \leq 0.05$. Given $\iiint_E f \, dV = 50$, bound $\iiint_E g \, dV$.

Tasks

1. Check consistency of the data.
2. Derive sharp bounds for the new integral.
3. Explain sharpness.

Question 10 – Solution

Strategy. Integrate pointwise inequalities and scale uniform error by volume.

Step 1: Consistency $24 \leq \iiint_E f \, dV \leq 84$, so 50 is possible.

Step 2: Perturbation From $-0.05 \leq g - f \leq 0.05$ and $\text{vol}(E) = 12$,

$$-0.6 \leq \iiint_E (g - f) \, dV \leq 0.6.$$

Therefore $\boxed{49.4 \leq \iiint_E g \, dV \leq 50.6}.$

Verification The endpoints occur for $g = f - 0.05$ and $g = f + 0.05$, so the supplied data guarantee no tighter interval.