

Question 9

For continuous f, g, h on their respective intervals, prove the product rule for $f(x)g(y)h(z)$ on $[a, b] \times [c, d] \times [e, k]$, then apply it to $(1+x)y^2e^z$ on $[0, 1] \times [-1, 1] \times [0, \ln 2]$.

Tasks

1. Prove factorization.
2. Evaluate the example.
3. Check the sign.

Question 9 – Solution

Strategy. Pull factors independent of the current integration variable outside each inner integral.

Step 1: Proof Successive integration gives $\iiint fgh \, dV = (\int_a^b f)(\int_c^d g)(\int_e^k h)$. Continuity guarantees the orders are valid.

Step 2: Example $\int_0^1 (1+x)dx = 3/2$, $\int_{-1}^1 y^2 dy = 2/3$, and $\int_0^{\ln 2} e^z dz = 1$. Their product is $\boxed{1}$.

Verification All factors are nonnegative on the box, matching the positive result.