

Question 8

Let $I(a) = \int_0^1 \int_0^2 \int_0^3 (a + x - y + z) \, dz \, dy \, dx$. Find a so the average integrand is 5.

Tasks

1. Evaluate $I(a)$.
2. Solve the average condition.
3. Verify using coordinate averages.

Question 8 – Solution

Strategy. Convert average 5 into total 5 times box volume.

Step 1: Integral The volume is 6 and the coordinate averages are $(1/2, 1, 3/2)$, hence $I(a) = 6(a + 1)$.

Step 2: Solve $I(a)/6 = 5$ gives $a + 1 = 5$, so $\boxed{a = 4}$.

Verification At $a = 4$, the average is $4 + 1/2 - 1 + 3/2 = 5$, and the total integral is 30.