

Question 7

For $E = \{x, y, z \geq 0 : x + y + z \leq 1\}$, rewrite $\iiint_E z \, dV$ in the orders $dz \, dy \, dx$ and $dx \, dz \, dy$, then evaluate.

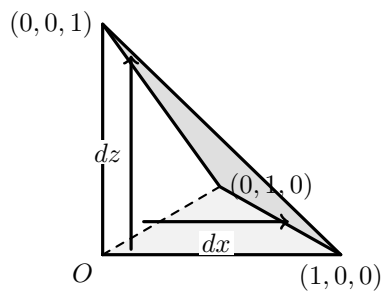
Tasks

1. Give both sets of bounds.
2. Evaluate one order.
3. Verify by symmetry.

Question 7 – Solution

Strategy. Each order peels coordinates from the simplex inequality.

Step 1: Geometry The simplex is bounded by the coordinate planes and $x + y + z = 1$. The two marked fibers show which coordinate is peeled off first.



Step 2: Orders $I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} z \, dz \, dy \, dx = \int_0^1 \int_0^{1-y} \int_0^{1-y-z} z \, dx \, dz \, dy$. The first form gives $I = \frac{1}{6} \int_0^1 (1-x)^3 dx = \boxed{\frac{1}{24}}$.

Verification The simplex volume is $1/6$ and its centroid has $\bar{z} = 1/4$, so the moment is $(1/6)(1/4) = 1/24$.