

Question 5

Find the average of $f = x^2 + y^2 + z^2$ on $[-1, 1]^3$.

Tasks

1. Use symmetry and separability.
2. Compute the average.
3. Compare with the range.

Question 5 – Solution

Strategy. The three squared-coordinate contributions are identical.

Step 1: Integral $\iiint_B x^2 dV = (\int_{-1}^1 x^2 dx)(2)(2) = 8/3$. Thus $\iiint_B f dV = 3(8/3) = 8$.

Step 2: Average Since $\text{vol}(B) = 8$, $\boxed{f_{\text{avg}} = 1}$.

Verification $0 \leq f \leq 3$, and each coordinate square has average $1/3$, so their sum has average 1 .