

Question 4

A box $0 \leq x \leq 2$, $0 \leq y \leq 1$, $0 \leq z \leq 3$ has density $\rho = 1 + x + z$.

Tasks

1. Find its mass.
2. Find the average density.
3. Check against the density range.

Question 4 – Solution

Strategy. Integrate density and divide by volume.

Step 1: Mass $M = \int_0^2 \int_0^1 \int_0^3 (1 + x + z) dz dy dx = \int_0^2 (15/2 + 3x) dx = \boxed{21}$.

Step 2: Average The volume is 6, so $\boxed{\rho_{\text{avg}} = 21/6 = 7/2}$.

Verification Here $1 \leq \rho \leq 6$; $7/2$ lies in this interval. Also the coordinate-average rule gives $1 + 1 + 3/2 = 7/2$.