

Question 3

Find the volume of the first-octant tetrahedron $x/2 + y/3 + z/6 \leq 1$.

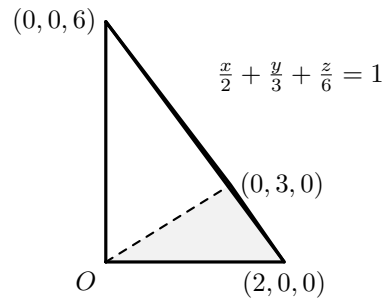
Tasks

1. Write bounds in the order $dz\,dy\,dx$.
2. Evaluate.
3. Verify geometrically.

Question 3 – Solution

Strategy. Solve the plane successively for z and for the projected y -boundary.

Step 1: Geometry The coordinate-plane faces meet the sloping face at the three axis intercepts shown.



Step 2: Bounds and integral $0 \leq x \leq 2$, $0 \leq y \leq 3 - 3x/2$, and $0 \leq z \leq 6 - 3x - 2y$. Hence $V = \int_0^2 \int_0^{3-3x/2} (6 - 3x - 2y) dy dx = \int_0^2 \frac{9}{4} (2 - x)^2 dx = \boxed{6}$.

Verification The intercept formula gives $V = (1/6)(2)(3)(6) = 6$.