

Question 2

Let $E = \{0 \leq x \leq 1, 0 \leq y \leq 1 - x, 0 \leq z \leq 2 - x - y\}$.

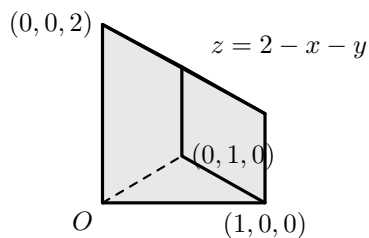
Tasks

1. Interpret the bounds.
2. Evaluate $\iiint_E 1 \, dV$.
3. Check positivity of every bound.

Question 2 – Solution

Strategy. Integrate vertical height over the triangular projection.

Step 1: Geometry The projection is the triangle $x \geq 0$, $y \geq 0$, $x + y \leq 1$. Above it, the solid runs from $z = 0$ to the sloping plane $z = 2 - x - y$.



Step 2: Evaluate $V = \int_0^1 \int_0^{1-x} \int_0^{2-x-y} 1 \, dz \, dy \, dx = \int_0^1 \left(\frac{3}{2} - 2x + \frac{x^2}{2} \right) dx = \boxed{\frac{2}{3}}.$

Verification On the projection, $2 - x - y \geq 1$, so every vertical bound is positive.