

Question 1

Evaluate $\iiint_B (x + 2y + 3z) \, dV$ on $B = [0, 1] \times [0, 2] \times [-1, 1]$.

Tasks

1. Write a Cartesian iterated integral.
2. Evaluate it.
3. Verify by average values.

Question 1 – Solution

Strategy. Integrate termwise over the box.

Step 1: Set up $I = \int_0^1 \int_0^2 \int_{-1}^1 (x + 2y + 3z) dz dy dx$.

Step 2: Evaluate The z term is odd; the other terms give $2x + 4y$. Thus $I = \int_0^1 \int_0^2 (2x + 4y) dy dx = \int_0^1 (4x + 8) dx = \boxed{10}$.

Verification The box volume is 4 and its coordinate averages are $(1/2, 1, 0)$, so the integrand average is $5/2$ and $4(5/2) = 10$.