

Question 9

An unknown annulus $a \leq r \leq b$, with $0 < a < b$, has area 3π and satisfies

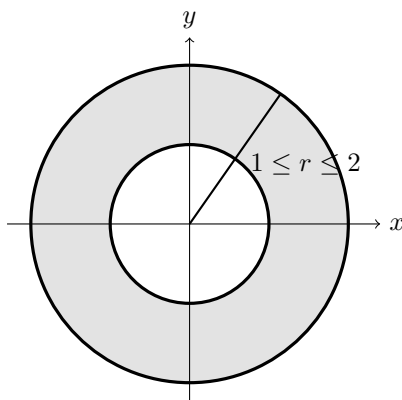
$$\iint_D (x^2 + y^2) dA = \frac{15\pi}{2}.$$

Tasks

1. Translate both conditions into equations for a^2, b^2 .
2. Recover the radii.
3. Verify both measurements.

Question 9 – Solution

Strategy. Set $A = a^2$ and $B = b^2$; area gives their difference, while the radial moment gives a difference of squares.



Step 1: Equations Area gives

$$\pi(b^2 - a^2) = 3\pi, \quad \text{so} \quad B - A = 3.$$

Also

$$\int_0^{2\pi} \int_a^b r^3 dr d\theta = \frac{\pi}{2}(b^4 - a^4) = \frac{15\pi}{2},$$

so $(B - A)(B + A) = 15$.

Step 2: Solve Since $B - A = 3$, we get $B + A = 5$. Therefore $B = 4$, $A = 1$, and positivity gives

$$\boxed{a = 1, \quad b = 2}.$$

Verification The area is $\pi(4 - 1) = 3\pi$, and the moment is $\frac{\pi}{2}(16 - 1) = 15\pi/2$. Both conditions hold exactly.