

Question 8

Find the area of one petal of the rose $r = \cos(2\theta)$ centered on the positive x -axis.

Tasks

1. Determine the angular interval for exactly that petal.
2. Set up and evaluate its area.
3. Explain why negative r values are excluded.

Question 8 – Solution

Strategy. The desired petal begins and ends where $\cos(2\theta) = 0$ around $\theta = 0$.

Step 1: Bounds The adjacent zeros are $\theta = \pm\pi/4$, and $\cos(2\theta) \geq 0$ between them. Thus $0 \leq r \leq \cos(2\theta)$.

Step 2: Area

$$\begin{aligned} A &= \int_{-\pi/4}^{\pi/4} \int_0^{\cos 2\theta} r \, dr \, d\theta = \frac{1}{2} \int_{-\pi/4}^{\pi/4} \cos^2(2\theta) d\theta \\ &= \frac{1}{4} \left[\theta + \frac{\sin 4\theta}{4} \right]_{-\pi/4}^{\pi/4} = \boxed{\frac{\pi}{8}}. \end{aligned}$$

Verification The sine endpoint terms vanish. Extending beyond these zeros introduces negative radial values, which trace different petals rather than enlarging the positive- x petal.