

Question 7

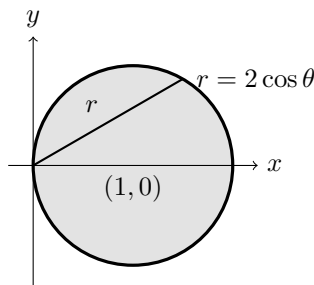
Let D be the disk $(x - 1)^2 + y^2 \leq 1$. Evaluate $\iint_D x \, dA$ in polar coordinates.

Tasks

1. Convert the offset circle to polar bounds.
2. Evaluate the integral.
3. Verify the result using area and symmetry.

Question 7 – Solution

Strategy. Rewrite the circle as $r = 2 \cos \theta$ and remember that $x = r \cos \theta$ contributes another factor of r with dA .



Step 1: Bounds The boundary equation becomes $r^2 - 2r \cos \theta = 0$. Thus

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq r \leq 2 \cos \theta.$$

Step 2: Evaluate

$$\begin{aligned} \iint_D x \, dA &= \int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} r^2 \cos \theta \, dr \, d\theta \\ &= \frac{8}{3} \int_{-\pi/2}^{\pi/2} \cos^4 \theta \, d\theta = \frac{8}{3} \cdot \frac{3\pi}{8} = \boxed{\pi}. \end{aligned}$$

Verification The disk has area π and is symmetric about the vertical line $x = 1$, so its average x -coordinate is 1. Hence its first moment is $1 \cdot \pi = \pi$.