

Question 6

Let D be the first-quadrant triangle $x \geq 0$, $y \geq 0$, $x + y \leq 2$.

Tasks

1. Convert the line boundary to a radial bound.
2. Use polar coordinates to compute the area.
3. Verify with elementary triangle geometry.

Question 6 – Solution

Strategy. A ray at angle θ meets $x + y = 2$ when $r(\cos \theta + \sin \theta) = 2$.

Step 1: Polar bounds First-quadrant rays give $0 \leq \theta \leq \pi/2$, and

$$0 \leq r \leq \frac{2}{\cos \theta + \sin \theta}.$$

Step 2: Area

$$A = \frac{1}{2} \int_0^{\pi/2} \frac{4}{(\cos \theta + \sin \theta)^2} d\theta.$$

Let $u = \tan \theta$. Since $(\cos \theta + \sin \theta)^2 = (1 + u)^2 / (1 + u^2)$ and $d\theta = du / (1 + u^2)$,

$$A = 2 \int_0^\infty \frac{du}{(1 + u)^2} = \boxed{2}.$$

Verification The Cartesian triangle has legs of length 2, so its area is $\frac{1}{2}(2)(2) = 2$. The nonconstant radial bound is essential; a constant bound would describe a sector, not the triangle.