

**Question 5**

For  $a > 0$ , evaluate

$$\iint_{x^2+y^2 \leq a^2} e^{-(x^2+y^2)} dA.$$

**Tasks**

1. Convert to polar coordinates.
2. Evaluate as a function of  $a$ .
3. Find the limit as  $a \rightarrow \infty$  and verify monotonicity.

**Question 5 – Solution**

**Strategy.** The exponent and disk are radial; use  $u = r^2$  after including the Jacobian.

**Step 1: Set up**

$$I(a) = \int_0^{2\pi} \int_0^a e^{-r^2} r \, dr \, d\theta.$$

**Step 2: Evaluate** With  $u = r^2$ ,  $du = 2r \, dr$ ,

$$I(a) = 2\pi \left[ -\frac{1}{2} e^{-r^2} \right]_0^a = \boxed{\pi(1 - e^{-a^2})}.$$

**Step 3: Checks** Since  $I'(a) = 2\pi a e^{-a^2} > 0$ , enlarging the disk increases the integral. Also

$$\boxed{\lim_{a \rightarrow \infty} I(a) = \pi}.$$

The result approaches a finite limit because the exponential decay dominates the growing circumference.