

Question 4

Find the area inside $r = 2 \cos \theta$ but outside the unit circle $r = 1$.

Tasks

1. Determine the angles where the radial bounds are ordered.
2. Set up and evaluate the area integral.
3. Interpret the boundary intersections.

Question 4 – Solution

Strategy. Solve $2 \cos \theta = 1$ to locate the two intersection rays, then integrate from the unit circle to the offset circle.

Step 1: Angles The curves meet when $\cos \theta = 1/2$, so $\theta = \pm\pi/3$. Between these angles, $2 \cos \theta \geq 1$.

Step 2: Area

$$\begin{aligned} A &= \int_{-\pi/3}^{\pi/3} \int_1^{2 \cos \theta} r \, dr \, d\theta \\ &= \frac{1}{2} \int_{-\pi/3}^{\pi/3} (4 \cos^2 \theta - 1) d\theta \\ &= \frac{1}{2} \left(4 \left(\frac{\pi}{3} + \frac{\sqrt{3}}{4} \right) - \frac{2\pi}{3} \right) = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}. \end{aligned}$$

Verification At both endpoint rays the inner and outer radii equal 1, so slice width vanishes. At $\theta = 0$ the slice runs from 1 to 2, confirming the order.