

Question 3

Find the area enclosed by the cardioid $r = 1 + \cos \theta$.

Tasks

1. Choose a complete nonduplicating angular interval.
2. Derive the polar area integral from a double integral.
3. Evaluate it exactly.

Question 3 – Solution

Strategy. Integrate 1 with radial bounds from the pole to the cardioid over one full turn.

Step 1: Bounds Since $1 + \cos \theta \geq 0$, one traversal is $0 \leq \theta \leq 2\pi$, $0 \leq r \leq 1 + \cos \theta$.

Step 2: Area integral

$$A = \int_0^{2\pi} \int_0^{1+\cos \theta} r \, dr \, d\theta = \frac{1}{2} \int_0^{2\pi} (1 + \cos \theta)^2 \, d\theta.$$

Step 3: Evaluate Over a full period, $\int \cos \theta = 0$ and $\int_0^{2\pi} \cos^2 \theta \, d\theta = \pi$. Thus

$$A = \frac{1}{2}(2\pi + 0 + \pi) = \boxed{\frac{3\pi}{2}}.$$

The radial bound vanishes only at $\theta = \pi$, so the interval neither omits nor duplicates positive-area portions.