

Question 2

Let D be the annular sector

$$1 \leq x^2 + y^2 \leq 9, \quad \frac{\pi}{6} \leq \arg(x + iy) \leq \frac{\pi}{2}.$$

Tasks

1. Write polar bounds.
2. Evaluate $\iint_D 1/\sqrt{x^2 + y^2} \, dA$.
3. Check finiteness and units.

Question 2 – Solution

Strategy. The factor $1/r$ cancels the polar Jacobian, leaving a rectangular integral in (r, θ) .

Step 1: Bounds The radial inequality gives $1 \leq r \leq 3$, while $\pi/6 \leq \theta \leq \pi/2$.

Step 2: Evaluate

$$\begin{aligned}\iint_D \frac{1}{\sqrt{x^2 + y^2}} dA &= \int_{\pi/6}^{\pi/2} \int_1^3 \frac{1}{r} (r \, dr \, d\theta) \\ &= \int_{\pi/6}^{\pi/2} 2 \, d\theta = \boxed{\frac{2\pi}{3}}.\end{aligned}$$

Verification The region stays at least one unit from the origin, so the integrand is bounded and no singularity occurs. After cancellation the radial length is 2 and the angular width is $\pi/3$.