

Question 1

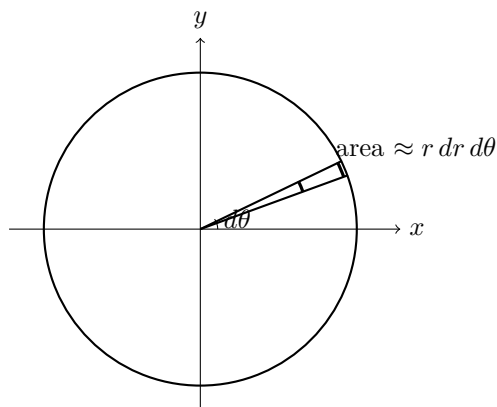
Let D be the disk $x^2 + y^2 \leq 4$.

Tasks

1. Convert $x^2 + y^2$ and dA to polar form.
2. Set up and evaluate $\iint_D (x^2 + y^2) dA$.
3. Explain the Jacobian factor geometrically.

Question 1 – Solution

Strategy. Radial symmetry turns the integrand into r^2 and the area element into $r \, dr \, d\theta$.



Step 1: Conversion The disk is $0 \leq r \leq 2$, $0 \leq \theta \leq 2\pi$, and $x^2 + y^2 = r^2$. Hence

$$\iint_D (x^2 + y^2) dA = \int_0^{2\pi} \int_0^2 r^2 (r \, dr \, d\theta).$$

Step 2: Evaluate

$$\int_0^{2\pi} \left[\frac{r^4}{4} \right]_0^2 d\theta = \int_0^{2\pi} 4 \, d\theta = \boxed{8\pi}.$$

Verification A thin polar cell has radial side dr and arc-length side approximately $r \, d\theta$, so its area is $r \, dr \, d\theta$. Omitting r would give the wrong units and value.