

Question 10

Let

$$D = \{(x, y) : 0 \leq y \leq 1, -y \leq x \leq y\}.$$

Tasks

1. Describe the region in the reverse order, including any required split.
2. Evaluate $\iint_D (x^2 + y) \, dA$ in the given order.
3. Use symmetry and the reversed description to check the structure of the answer.

Question 10 – Solution

Strategy. Use horizontal symmetry about the y -axis for evaluation, then express the lower boundary as $y = |x|$ when reversing.

Step 1: Reverse description The triangle has vertices $(0, 0)$, $(-1, 1)$, $(1, 1)$. In one compact form,

$$-1 \leq x \leq 1, \quad |x| \leq y \leq 1.$$

Without absolute values it splits into $-1 \leq x \leq 0$, $-x \leq y \leq 1$, and $0 \leq x \leq 1$, $x \leq y \leq 1$.

Step 2: Given order

$$\begin{aligned} \iint_D (x^2 + y) dA &= \int_0^1 \int_{-y}^y (x^2 + y) dx dy \\ &= \int_0^1 \left(\frac{2y^3}{3} + 2y^2 \right) dy \\ &= \frac{1}{6} + \frac{2}{3} = \boxed{\frac{5}{6}}. \end{aligned}$$

Step 3: Structural check In the reversed order, the two x -halves contribute equally because $x^2 + y$ is even in x . Thus

$$\iint_D (x^2 + y) dA = 2 \int_0^1 \int_x^1 (x^2 + y) dy dx.$$

Evaluating gives $2(5/12) = 5/6$. This verifies both the factor of two and the lower boundary $y = |x|$.