

Question 9

Let

$$D = \{(x, y) : 0 \leq x \leq 2, 0 \leq y \leq \min(x, 2 - x)\}.$$

Tasks

1. Explain why the vertical-slice description must split at $x = 1$.
2. Give a single Type II description of D .
3. Evaluate $\iint_D (x + 2y) \, dA$ using the more efficient description.

Question 9 – Solution

Strategy. Recognize the triangle with vertices $(0, 0)$, $(1, 1)$, $(2, 0)$; horizontal slices use one left and one right boundary.

Step 1: Vertical split The smaller upper boundary is x on $[0, 1]$ and $2 - x$ on $[1, 2]$. Thus vertical integration would require

$$\int_0^1 \int_0^x (\cdots) dy dx + \int_1^2 \int_0^{2-x} (\cdots) dy dx.$$

Step 2: Single horizontal description For $0 \leq y \leq 1$, the line $y = x$ gives $x = y$ and $y = 2 - x$ gives $x = 2 - y$. Hence

$$D = \{(x, y) : 0 \leq y \leq 1, y \leq x \leq 2 - y\}.$$

Step 3: Evaluate

$$\begin{aligned} \iint_D (x + 2y) dA &= \int_0^1 \int_y^{2-y} (x + 2y) dx dy \\ &= \int_0^1 \left[\frac{x^2}{2} + 2yx \right]_y^{2-y} dy \\ &= \int_0^1 (2 + 2y - 4y^2) dy = \left[2y + y^2 - \frac{4}{3}y^3 \right]_0^1 \\ &= \boxed{\frac{5}{3}}. \end{aligned}$$

The integrand is nonnegative on D , agreeing with the sign.