

Question 6

Consider

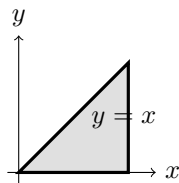
$$I = \int_0^1 \int_y^1 e^{x^2} dx dy.$$

Tasks

1. Describe the triangular region represented by the bounds.
2. Explain why the displayed order is not convenient for elementary integration.
3. Reverse the order and evaluate I exactly.

Question 6 – Solution

Strategy. Draw the triangle $0 \leq y \leq x \leq 1$; reversing the order creates an inner interval whose length supplies the factor needed for substitution.



Step 1: Region The displayed bounds say $0 \leq y \leq 1$ and $y \leq x \leq 1$. Equivalently,

$$D = \{(x, y) : 0 \leq x \leq 1, 0 \leq y \leq x\}.$$

Step 2: Why reverse In the given order, the inner antiderivative $\int e^{x^2} dx$ is not elementary. With dy first, e^{x^2} is constant in the inner variable.

Step 3: Reversed evaluation

$$\begin{aligned} I &= \int_0^1 \int_0^x e^{x^2} dy dx = \int_0^1 x e^{x^2} dx \\ &= \frac{1}{2} \int_0^1 e^u du = \left[\frac{e-1}{2} \right], \quad u = x^2. \end{aligned}$$

The integrand and region are positive, so the positive result is consistent. Differentiating $e^{x^2}/2$ returns $x e^{x^2}$.