

Question 5

Let

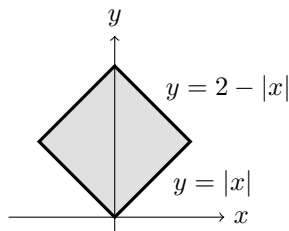
$$D = \{(x, y) : -1 \leq x \leq 1, |x| \leq y \leq 2 - |x|\}.$$

Tasks

1. Sketch or describe the region and identify its symmetry.
2. Evaluate $\iint_D y \, dA$ by splitting at $x = 0$ or using $|x|$ directly.
3. Find the area of D and hence the average value of y .

Question 5 – Solution

Strategy. Each vertical slice has endpoints symmetric about the horizontal line $y = 1$, making its average height immediate.



Step 1: Geometry The vertices are $(0, 0), (1, 1), (0, 2), (-1, 1)$. The diamond is symmetric about both $x = 0$ and $y = 1$.

Step 2: Integral of y

$$\begin{aligned}\iint_D y \, dA &= \int_{-1}^1 \int_{|x|}^{2-|x|} y \, dy \, dx \\ &= \frac{1}{2} \int_{-1}^1 ((2 - |x|)^2 - |x|^2) \, dx \\ &= \int_{-1}^1 (2 - 2|x|) \, dx = \boxed{2}.\end{aligned}$$

Step 3: Area and average The slice width is $2 - 2|x|$, so

$$A = \int_{-1}^1 (2 - 2|x|) \, dx = 2.$$

Therefore $y_{\text{avg}} = 2/2 = \boxed{1}$, exactly the symmetry line. This independently checks the integral: total equals average times area.