

Question 4

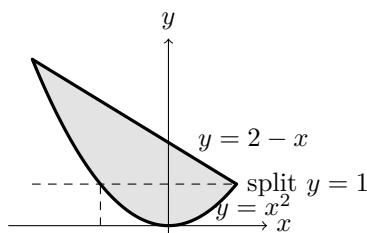
Let D be the region between $y = x^2$ and $y = 2 - x$.

Tasks

1. Describe D with vertical slices.
2. Reverse the order, explaining why horizontal slices require two integrals.
3. Compute the area in the reversed order and check it in the original order.

Question 4 – Solution

Strategy. Locate the intersection points and the height where the right boundary changes from the parabola to the line.



Step 1: Vertical description Solving $x^2 = 2 - x$ gives $x = -2, 1$. Hence

$$D = \{(x, y) : -2 \leq x \leq 1, x^2 \leq y \leq 2 - x\}.$$

Step 2: Reverse and split Horizontal slices begin at $x = -\sqrt{y}$. For $0 \leq y \leq 1$, they end at $x = \sqrt{y}$; for $1 \leq y \leq 4$, they end on the line $x = 2 - y$. Thus

$$\iint_D 1 \, dA = \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} 1 \, dx \, dy + \int_1^4 \int_{-\sqrt{y}}^{2-y} 1 \, dx \, dy.$$

Step 3: Area

$$\begin{aligned} A &= \int_0^1 2\sqrt{y} \, dy + \int_1^4 (2 - y + \sqrt{y}) \, dy \\ &= \frac{4}{3} + \left[2y - \frac{y^2}{2} + \frac{2}{3}y^{3/2} \right]_1^4 = \frac{4}{3} + \frac{19}{6} = \boxed{\frac{9}{2}}. \end{aligned}$$

The original order gives $\int_{-2}^1 (2 - x - x^2) dx = 9/2$, confirming the split.