

Question 3

The region D lies between $y = x$ and $y = \sqrt{x}$ in the first quadrant.

Tasks

1. Describe D using vertical slices and using horizontal slices.
2. Write $\iint_D 1 \, dA$ in both orders.
3. Evaluate the area and verify the two descriptions agree.

Question 3 – Solution

Strategy. Solve each boundary for the inner variable and use the common endpoints $(0, 0)$ and $(1, 1)$.

Step 1: Two descriptions Since $x \leq \sqrt{x}$ on $[0, 1]$,

$$D = \{0 \leq x \leq 1, x \leq y \leq \sqrt{x}\}.$$

Solving for x , the parabola is $x = y^2$ and the line is $x = y$. For $0 \leq y \leq 1$, $y^2 \leq y$, so

$$D = \{0 \leq y \leq 1, y^2 \leq x \leq y\}.$$

Step 2: Both orders

$$\iint_D 1 \, dA = \int_0^1 \int_x^{\sqrt{x}} 1 \, dy \, dx = \int_0^1 \int_{y^2}^y 1 \, dx \, dy.$$

Step 3: Area The horizontal description gives

$$\text{area}(D) = \int_0^1 (y - y^2) \, dy = \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1 = \boxed{\frac{1}{6}}.$$

Directly, $\int_0^1 (\sqrt{x} - x) \, dx = 2/3 - 1/2 = 1/6$, verifying the reversal.