

Question 2

Let D be bounded by $x = y^2$ and $x = 2y$.

Tasks

1. Describe D as a Type II region.
2. Evaluate $\iint_D x \, dA$ using horizontal slices.
3. Check the sign and the intersection endpoints.

Question 2 – Solution

Strategy. Treat x as the inner variable and determine which curve is leftmost between the intersection values of y .

Step 1: Region From $y^2 = 2y$, the curves meet at $y = 0, 2$. On this interval, $y^2 \leq 2y$, so

$$D = \{(x, y) : 0 \leq y \leq 2, y^2 \leq x \leq 2y\}.$$

Step 2: Integral

$$\begin{aligned}\iint_D x \, dA &= \int_0^2 \int_{y^2}^{2y} x \, dx \, dy = \frac{1}{2} \int_0^2 (4y^2 - y^4) \, dy \\ &= \left[\frac{2y^3}{3} - \frac{y^5}{5} \right]_0^2 = \frac{16}{3} - \frac{16}{5} = \boxed{\frac{32}{15}}.\end{aligned}$$

Verification Both boundaries have $x \geq 0$ on $0 \leq y \leq 2$, so the integral of x must be nonnegative. The width $2y - y^2 = y(2 - y)$ vanishes at both intersection endpoints as the geometry requires.