

Question 1

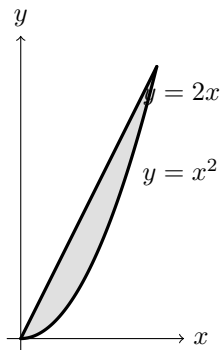
Let D be bounded by $y = x^2$ and $y = 2x$.

Tasks

1. Find the intersection points and describe D as a Type I region.
2. Set up and evaluate $\iint_D (x + y) dA$.
3. Verify that the bounds keep the lower curve below the upper curve.

Question 1 – Solution

Strategy. Solve the boundary intersection first, then use vertical slices from the parabola to the line.



Step 1: Region The curves meet where $x^2 = 2x$, so $x = 0, 2$. For $0 \leq x \leq 2$, $x^2 \leq 2x$. Thus

$$D = \{(x, y) : 0 \leq x \leq 2, x^2 \leq y \leq 2x\}.$$

Step 2: Integral

$$\begin{aligned} \iint_D (x + y) \, dA &= \int_0^2 \int_{x^2}^{2x} (x + y) \, dy \, dx \\ &= \int_0^2 \left[xy + \frac{y^2}{2} \right]_{x^2}^{2x} dx \\ &= \int_0^2 \left(4x^2 - x^3 - \frac{x^4}{2} \right) dx \\ &= \frac{32}{3} - 4 - \frac{16}{5} = \boxed{\frac{52}{15}}. \end{aligned}$$

Verification The slice height is $2x - x^2 = x(2 - x) \geq 0$ on $[0, 2]$. Also $x + y \geq 0$ throughout D , consistent with the positive answer.