

Question 7

A temperature field on a rectangular plate is

$$T(x, y) = 18 + 2x + y^2, \quad 0 \leq x \leq 3, \quad -1 \leq y \leq 1.$$

Tasks

1. Compute the average temperature using an iterated integral.
2. Compare it with the minimum and maximum temperatures on the plate.
3. Exhibit two distinct points where the temperature equals its average.

Question 7 – Solution

Strategy. Integrate the three terms separately, divide by plate area, and then solve the resulting level equation at convenient choices of y .

Step 1: Total and average The plate area is $A = 3 \cdot 2 = 6$. Its total temperature integral is

$$\begin{aligned}\int_0^3 \int_{-1}^1 (18 + 2x + y^2) dy dx &= 18(6) + \left(\int_0^3 2x dx \right) (2) + 3 \int_{-1}^1 y^2 dy \\ &= 108 + 18 + 2 = 128.\end{aligned}$$

Hence

$$T_{\text{avg}} = \frac{128}{6} = \frac{64}{3}.$$

Step 2: Range check The minimum is $T(0, 0) = 18$. The maximum 25 occurs at $(3, 1)$ and $(3, -1)$. Since

$$18 < \frac{64}{3} < 25,$$

the average lies strictly inside the temperature range, as required.

Step 3: Points at the average Solve

$$18 + 2x + y^2 = \frac{64}{3}.$$

Taking $y = 0$ gives $x = 5/3$, while taking $y = 1$ gives $x = 7/6$. Thus two examples are

$$\left(\frac{5}{3}, 0 \right) \quad \text{and} \quad \left(\frac{7}{6}, 1 \right).$$

Both lie on the plate, and substitution gives $64/3$ at each.