

Question 5

Evaluate

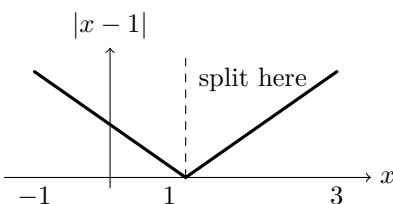
$$\iint_R |x - 1|(1 + y) \, dA, \quad R = [-1, 3] \times [0, 2].$$

Tasks

1. Choose an efficient order of integration.
2. Split at the correct location to remove the absolute value.
3. Evaluate the integral and its average value over R .

Question 5 – Solution

Strategy. Integrate the positive y -factor first, then split the remaining x -integral where $x - 1$ changes sign.



Step 1: Inner integral Using dy first,

$$\begin{aligned}\int_{-1}^3 \int_0^2 |x - 1|(1 + y) dy dx &= \int_{-1}^3 |x - 1| \left[y + \frac{y^2}{2} \right]_0^2 dx \\ &= 4 \int_{-1}^3 |x - 1| dx.\end{aligned}$$

Step 2: Absolute-value split

$$\begin{aligned}\int_{-1}^3 |x - 1| dx &= \int_{-1}^1 (1 - x) dx + \int_1^3 (x - 1) dx \\ &= 2 + 2 = 4.\end{aligned}$$

Therefore

$$\boxed{\iint_R |x - 1|(1 + y) dA = 16}.$$

Step 3: Average and check The rectangle has area $4 \cdot 2 = 8$, so

$$\boxed{f_{\text{avg}} = \frac{16}{8} = 2}.$$

The two x -pieces are mirror images about $x = 1$, and the nonnegative integrand confirms the positive sign.