

Question 4

A student is given

$$I = \int_{-1}^1 \int_0^2 (x - 2y) \, dx \, dy$$

and claims the inner integral is $[x^2/2 - 2xy]_{-1}^1$.

Tasks

1. Diagnose the student's error using the differential and bounds.
2. Evaluate the integral correctly as written.
3. Reverse the order on the same rectangle and verify the result.

Question 4 – Solution

Strategy. Read an iterated integral from the inside out: the inner differential determines both the variable of integration and which pair of bounds belongs to it.

Step 1: Diagnose The inner differential is dx , so x runs from 0 to 2 while y is held constant. The student incorrectly used the outer y -bounds, -1 and 1 , as x -bounds.

Step 2: Evaluate as written

$$\begin{aligned} I &= \int_{-1}^1 \left[\frac{x^2}{2} - 2yx \right]_0^2 dy = \int_{-1}^1 (2 - 4y) dy \\ &= [2y - 2y^2]_{-1}^1 = \boxed{4}. \end{aligned}$$

The odd contribution $-4y$ cancels over $[-1, 1]$.

Step 3: Reverse correctly Constant rectangular bounds reverse to

$$I = \int_0^2 \int_{-1}^1 (x - 2y) dy dx.$$

Then

$$I = \int_0^2 [xy - y^2]_{-1}^1 dx = \int_0^2 2x dx = [x^2]_0^2 = 4.$$

The agreement verifies both the order reversal and the association of each bound pair with its variable.