

Question 3

Consider

$$I = \int_0^1 \int_0^2 x e^{xy} dy dx.$$

Tasks

1. Evaluate I using the displayed order.
2. Write the reversed iterated integral and its inner antiderivative for $y \neq 0$.
3. Explain why the displayed order is substantially more efficient and check the behavior at $x = 0$.

Question 3 – Solution

Strategy. Treat x as constant in the inner integral; the factor x is exactly the derivative needed for e^{xy} with respect to y .

Step 1: Efficient order For $x \neq 0$,

$$\int_0^2 x e^{xy} dy = [e^{xy}]_0^2 = e^{2x} - 1.$$

At $x = 0$ the original integrand is identically zero and the same expression gives $e^0 - 1 = 0$, so the formula extends continuously. Hence

$$\begin{aligned} I &= \int_0^1 (e^{2x} - 1) dx = \left[\frac{1}{2} e^{2x} - x \right]_0^1 \\ &= \boxed{\frac{e^2 - 3}{2}}. \end{aligned}$$

Step 2: Reversed order The rectangle permits

$$I = \int_0^2 \int_0^1 x e^{xy} dx dy.$$

For $y \neq 0$, integration by parts gives

$$\int x e^{xy} dx = \frac{e^{xy}(xy - 1)}{y^2} + C,$$

so the inner result is

$$\frac{e^y(y - 1) + 1}{y^2}.$$

Step 3: Efficiency and verification The reversed expression has a removable singular appearance at $y = 0$ and requires another nonobvious antiderivative. The original order collapses immediately by substitution. Continuity of $x e^{xy}$ on the rectangle guarantees by Fubini's theorem that either completed order has the same value.